

# A Tri-objective model for multi-product multi-period inventory planning with substitutable goods and random demand

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**Abstract** The performed studies show that considering substitution goods can be profitable. The substitution is replacing one product with another in an inventory system. When a product has a shortage, a certain percentage of its demand can be replaced with similar goods. In this paper, a Tri-objective model with the substitution assumptions is considered. In here, the substitution means that there is a relation between different items that allows these items to be used instead of each other. Demands are considered probabilistic and there are some other assumptions as follows: planning is multi-period and multi product, inventory control parameters are fixed during the planning period. The shortage is allowed, but in the form of lost sale, the inventory of the beginning of the first period is very few (almost zero) and the remaining inventory at the end of each period will be moved to the next period. Objective functions are looking for maximizing the profit, minimizing the risk of facing slack and minimizing of the dissatisfaction arising from the substitution. Model is solved with two approaches: first with the LP-Metric method and next by two meta-heuristic algorithms such as NSGA-II and Differential Evolution. Most researches have focused on profit maximization or costs minimization. The current paper considers a multi-product and multi-period triple-objective model. The goods may be substituted with similar ones. The results of solving the model indicate that if there is a relation between the products items, considering this relationship in modeling, will lead to improved results. A part of this improvement is a result of reduced maintenance cost. With the substitution of items, we can both increase our profits and sell items that their expiry date is near to finish (arrangement type of substitution) and avoid loses.

**Keyword:** Inventory Planning, Substitute Products, Multi-Product, Multi-Period, Meta-Heuristic.

## 1 Introduction

In studies performed so far, this has been proved that considering substitution problem results in improved profitability of the collection [1-3], also in solving large-scale requirements planning problems with component substitution options paper refers that in these problems, also, considering the substitution issue leads to better results [4].

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Regarding the substitution problem, it can be said when a company performs reproducing in addition to the production, one-way substitution can occur from the manufacturer side [5, 6].

Substitution in a two-products supply chain is also discussed [7].

Substitution in simultaneous order of two products model [8] and economic order quantity with demand dependent on inventory, are also studied for two products [9].

Unilateral substitution in a periodic review model, with two items, is also discussed, where the objective of the model is to minimize the expected costs during the planning period [10].

A limited inventory control and multi-products, pricing model is studied about perishable products, considering these three connections: substitution, supplementary or irrelevant, with the aim of finding the price and optimal quantity, which maximizes total profit [11].

Also, if we consider other aspects of the model such as being multi-objective, multi-period and multi-product, studies with some similar assumptions can be mentioned such as a model for multi-product and multi-period planning, assuming discount and inflation [12-14], a model to determine the order optimal quantity and reorder point with the aim of maximizing profit [15], single-product possible inventory control problem with two modes of fast ordering and ordering with a period delivery time [16], a two-objective model that one of the objectives seeks to minimize the costs and the other maximizes service level [17], multi-product inventory system with quick response and capital constraint involved in the inventory, which inventory items demand are dependent in it [18].

Akcan presented a new approximation for inventory control system with decision variable lead time and stochastic demand [19]. Vaziri et.al [20], researched with the title "An integrated production and procurement design for a multi-period multi-product manufacturing system with machine assignment and warehouse constraint".

They proposed a production-procurement plan that integrates EOQ with EPQ for a multi-period multi-product production-inventory system with a restrict ware-house capacity [20-23].

Seda Turk et al. [24] presented a two-stage integrated approach to the inventory planning and supplier selection. In the following stage, an inventory model is created.

They assess the performance of three MOEAs with tuned parameters, namely NSGA-II, SPEA2 and IBEA. All in all, NSGA-II is the best performance MOEA product high quality trade-off solutions to the united problem of supplier selection and inventory planning [24].

S.M.J. MirzapourAl-e-hashem and YacineRekik inscribe a multi-product multi-period Inventory Routing Problem (IRP Where Vehicles with multiple capacitated spread products from multiple suppliers to a single plant to show the given demand of each product over a limited planning perspective [25].

Seyed Mohsen Mousavi and Seyed Hamidreza Pasandideh considered a limited horizon multi product and multi period economic order quantity like seasonal items that the rate of demand is determined but variable in each period.

They present modeling technique for all units' discount (AUD) policy and mix binary integer programming. Controlling the inventory costs system divided in three parts: ordering cost, holding cost, and purchase cost and solve the proposed model, with genetic algorithm [26].

Zhixue Liao et al. presented a fuzzy modeling to find the coefficient for correlation with an approximation approach for multi-period and multi-products [27].

In this paper, the researchers make use of optimization methods, to improve performance of the heuristic engine to solve "single period multi-product inventory problem" (Uday et 2004). Lakdere Benkherouf et al. proposed a finite horizon inventory control problem for two

substitutable products and demands are time-varying. Numerical examples presented for the optimal ordering schedule to minimize the total cost [28]. Ramyal et al. present a model in supply chain to reduce the total cost involving inventory costs, manufacturing costs, work force costs, hiring and rising costs and also to optimize the minimum of supplier reliability and improve the system performance.

The results show the robustness of the proposed algorithm to examine the Pareto solutions [29].

Barinci et al. [30], introduced a sectoral supply functions approach of equilibrium dynamics in the context of a simple model of overlapping generations with heterogeneous goods. Alizadeh Foroutan et al. [31], studied the green vehicle routing and scheduling problem with heterogeneous fleet, including reverse logistics in the form of collecting returned goods along with weighted earliness and tardiness costs to establish a trade-off between operational and environmental costs and to minimize both simultaneously.

In this regard, a mixed integer non-linear programming (MINLP) model is proposed.

In the second section of the study, some necessary definitions are presented. The model and its description are provided in the third section.

In the next section, the performance of the model was studied with numerical examples.

The fifth section discusses the second goal function. Finally, the results of this research are presented in the sixth section.

## 2 Definitions

**2.1 Substitution:** is the possibility of replacing one product with another in an inventory system. When a product faces shortage, a certain percentage of its demand can be replaced with similar goods [9].

**2.2 The main demand of a product:** is the amount of demand for a product which isn't occurred by replacement, but is the direct demand of that product [2].

**2.3 Substitution demands:** is the amount of demand for a product which occurs due to the lack of another product, to replace the original product with this one [2].

**2.4 Total demand:** is sum of original demand and substitution of a product [2].

## 3 The proposed model

In similar articles, planning is done for one period [2], or the demand is not probable, or planning is performed for maximum two similar products [8-10]. In cases where demand is possible and planning is multi-product and multi-period [1-3], there is only one objective function which focuses on profit maximization or costs minimization. The present article focuses on a multi-product and multi-period triple-objective model, assuming substitution existence and demand possibility, which in addition to extensive covering of items in the similar articles, is also triple-objective. According to our researches there was no similar case, up to the time of writing this paper.

### 3.1 Model assumption

- Demand is uniformly distributed in each period.
- Maintenance cost is fixed in different periods.
- Items price is fixed in different periods.
- Substitution relationship exists between some inventory items.
- The shortage is allowed, and is constant in different periods in the form of lost sale and shortage price.
- The amount of inventory is insignificant at the beginning of the study, and the remaining items are transferred to the next at the end of each period.
- The duration of all of the periods is the same.

Schneider, in his article titled Level of Service (LOS) in inventory control systems, explains that in periodic review systems, order to a certain degree of inventory leads to better results than the economic size of order [21]. Because periodic review is considered in this model, so we use order to a certain level of inventory approach.

### 3.2 Parameters and Variables

$a_{jt}$  = Parameter for demand of product  $j^{\text{th}}$  in period  $t^{\text{th}}$

$c_{jt}$  = Parameter for demand of product  $j^{\text{th}}$  in period  $t^{\text{th}}$

$m$  = Number of periods

$n$  = Number of inventory items

$ln_{jt}^{\text{first}}$  = Beginning inventory of product  $j^{\text{th}}$  in period  $t^{\text{th}}$

$Q_{jt}$  = Order amount of the product  $j^{\text{th}}$  in period  $t^{\text{th}}$

$S_j$  = Shortage price of each product unit  $j^{\text{th}}$  per time unit

$oc_j$  = Cost of product  $i^{\text{th}}$  order

$h_j$  = Maintenance cost of each product unit  $j^{\text{th}}$  per time unit

**Bignum** = A very large number.

**Capital** = Maximum allocation of capital.

**Space** = Maximum allocation of space.

**Cost<sub>j</sub>** = purchase price of each product unit  $j^{\text{th}}$

**pr<sub>j</sub>** = sale price of each product  $j^{\text{th}}$ .

**Volume<sub>j</sub>** = space occupied by each product unit  $j^{\text{th}}$

$Q_j^{\text{max}}$  = maximum  $j^{\text{th}}$  inventory levels (problem variable)

$\alpha_{ij}$  = Substitution rate of product  $i^{\text{th}}$  instead of product  $j^{\text{th}}$  ( $\alpha_{jj} = 1$  and  $\alpha_{ij} \leq 1$ )

$y_{ijt}$  = Number of items of inventory  $i^{\text{th}}$  that is allocated to product demand  $j^{\text{th}}$  in period  $t^{\text{th}}$ .  
(Problem Variable)

### 4 The model

Consider the following Classic objective function:

$$\begin{aligned}
Maxz_1 = & \sum_{t=1}^m \sum_{j=1}^n \Pr_j \left( \int_{r_{jt}=0}^{Q_{jt}^{\max}} r_{jt} \cdot fr_{jt} \cdot dr_{jt} + \int_{Q_{jt}^{\max}}^{\infty} Q_{jt}^{\max} \cdot fr_{jt} \cdot dr_{jt} \right) \\
& - \sum_{t=1}^m \sum_{j=1}^n h_j \cdot \int_{r_{jt}=0}^{Q_{jt}^{\max}} (Q_{jt}^{\max} - r_{jt}) \cdot fr_{jt} \cdot dr_{jt} \\
& - \sum_{t=1}^m \sum_{j=1}^n s_j \cdot \int_{r_{jt}=Q_{jt}^{\max}}^{\infty} (r_{jt} - Q_{jt}^{\max}) \cdot fr_{jt} \cdot dr_{jt} \\
& - m \cdot \sum_{j=1}^n oc_j - \sum_{t=1}^m \sum_{j=1}^n cost_j \cdot Q_{jt}
\end{aligned}$$

In a system of periodic review, using this function, the maximum level of inventory is determined in a way that maximizes the system profit.

In the income sector, if the maximum level of inventory is unlimited ( $Q_{jt}^{\max} = \infty$ ) we don't need to divide integral to two pieces and the result of integral will be equal to the mathematical expectancy sale. In substitution mode, with few ignorance, maximum inventory level can be considered unlimited, so we can say that the average sale of the product is equal to the expected sales level. In other words, we have:

$$\sum_{i=1}^n y_{ijt} \leq E(r_{jt}) \quad (1)$$

Given the upper condition in model constraints, income can be calculated according to (2).

$$\sum_{t=1}^m \sum_{j=1}^n \sum_{i=1}^n (\Pr_i \cdot y_{ijt}) \quad (2)$$

Assuming the substitution occurrence in the system, then the maximum actual inventory level is:

$$Q_j^{\max} - \sum_{\substack{i=1 \\ i \neq j}}^n y_{jit} + \sum_{\substack{i=1 \\ i \neq j}}^n y_{ijt} \quad (3)$$

However, considering this maximum inventory levels and lower or upper limits for demand, maintenance cost and shortage are rewritten as (4) and (5). The remaining objective function is also used with the same form.

$$\sum_{t=1}^m \sum_{j=1}^n h_j \cdot \int_{r_{jt}=a_{jt}}^{Q_j^{\max} - \sum_{\substack{i=1 \\ i \neq j}}^n y_{jit} + \sum_{\substack{i=1 \\ i \neq j}}^n y_{ijt}} ((Q_j^{\max} - \sum_{\substack{i=1 \\ i \neq j}}^n y_{jit} + \sum_{\substack{i=1 \\ i \neq j}}^n y_{ijt}) - r_{jt}) \cdot fr_{jt} \cdot dr_{jt} \quad (4)$$

$$\sum_{t=1}^m \sum_{j=1}^n s_j \cdot \int_{r_{jt}=Q_j^{\max} - \sum_{\substack{i=1 \\ i \neq j}}^n y_{jit} + \sum_{\substack{i=1 \\ i \neq j}}^n y_{ijt}}^{c_{jt}} (r_{jt} - (Q_j^{\max} - \sum_{\substack{i=1 \\ i \neq j}}^n y_{jit} + \sum_{\substack{i=1 \\ i \neq j}}^n y_{ijt})) \cdot fr_{jt} \cdot dr_{jt} \quad (5)$$

Thus, the classical objective function is rewritten with the mentioned changes and will be used as the first objective function.

In the obtained inventory models, the following integral indicates the possibility of facing with shortage.

$$\int_{Q^{\max}}^{\infty} fr.dr \quad (6)$$

Using this formula, and considering the expression (3) and the upper limit of demand, the second objective function is written according to (7).

$$Min_{z_2} = \sum_{t=1}^m \sum_{j=1}^n \int_{r_{jt}=Q_j^{\max} - \sum_{i=1, i \neq j}^n y_{jit} + \sum_{i=1, i \neq j}^n y_{ijt}}^{c_{jt}} fr_{jt}.dr_{jt} \quad (7)$$

About the third objective function, it should be said if parameter  $\alpha_{ij}$  shows the similarity of product  $i^{\text{th}}$  and  $j^{\text{th}}$ , the dissatisfaction  $1-\alpha_{ij}$  will be created due to the substitution of product  $i^{\text{th}}$  instead of product  $j^{\text{th}}$ . So the total dissatisfaction caused by the substitution of product  $i^{\text{th}}$  instead of product  $j^{\text{th}}$  in all periods for all products that need to be minimized (third objective function) will be equal to:

$$Min_{z_3} = \sum_{t=1}^m \sum_{j=1}^n \sum_{i=1}^n ((1-\alpha_{ij}).y_{ijt}) \quad (8)$$

#### 4.1 The constraints of model

Constraint (9): meeting the minimum demand

$$\alpha_{jt} \leq \sum_{i=1}^n y_{ijt} \quad j=1, \dots, n \quad t=1, \dots, m \quad (9)$$

Constraint (10): supply restriction

$$\sum_{i=1}^n y_{jit} \leq Q_j^{\max} \quad j=1, \dots, n \quad t=1, \dots, m \quad (10)$$

Constraint (11): Average sale

$$\sum_{i=1}^n y_{ijt} \leq E(r_{jt}) \quad j=1, \dots, n \quad t=1, \dots, m \quad (11)$$

Constraint (12): Minimum actual inventory level

$$Q_j^{\max} - \sum_{i=1, i \neq j}^n y_{jit} + \sum_{i=1, i \neq j}^n y_{ijt} \geq a_{jt} \quad j=1, \dots, n \quad t=1, \dots, m \quad (12)$$

Constraint (13): Maximum actual inventory level

$$Q_j^{\max} - \sum_{i=1, i \neq j}^n y_{jit} + \sum_{i=1, i \neq j}^n y_{ijt} \leq c_{jt} \quad j=1, \dots, n \quad t=1, \dots, m \quad (13)$$

Constraint (14): Beginning period inventory of the first period

$$In_{j1}^{\text{first}} = 0 \quad j=1, \dots, n \quad (14)$$

Constraint (15): Order quantity for each period

$$Q_{jt} = \text{Max}(Q_j^{\max} - In_{jt}^{\text{first}}, 0) \quad j=1, \dots, n \quad t=1, \dots, m \quad (15)$$

Constraint (16): Beginning period inventory of each period

$$In_{jt}^{first} = In_{jt-1}^{first} + Q_{jt-1} - \sum_{i=1}^n y_{jit-1} \quad j = 1, \dots, n \quad t = 2, \dots, m \quad (16)$$

Constraint (17): Main constraints involved in the inventory

$$\sum_{j=1}^n cost_j Q_j^{max} \leq Capital \quad (17)$$

constraint (18): Limited storage space

$$\sum_{j=1}^n volume_j Q_j^{max} \leq Space \quad (18)$$

Constraint (19): Lack of allocation of dissimilar goods limitation

$$\sum_{i=1}^n (y_{ijt} - Bignum \times \alpha_{ij}) \leq 0 \quad j = 1, \dots, n \quad t = 1, \dots, m \quad (19)$$

Constraint (20): Producing integer answers

$$y_{ijt}, Q_j^{max} = \text{integer} \quad i = 1, \dots, n \quad j = 1, \dots, n \quad t = 1, \dots, m \quad (20)$$

## 5 Numerical Examples

### 5.1 Solving problem with LP-Metric approach

The small-scale problems can be solved by GAMS software. Next, the obtained results can be used in LP-Metric method (table (1-4)).

### 5.2 Solving the model with Meta-heuristic algorithms

Due to the large number of variables, we are not able to use Lingo / Gams software. Table (4-2) shows the changes of time solution by increasing the number of periods and products. As you can see, the time solution is sensitive to period duration more than the number of products.

### 5.3 Production and interpretation of the initial answer

The primary answer of vector problem includes random values  $Q_j^{max}$ . The values produced for  $Q_j^{max}$  do not violate model constraints. It should be noted that the mechanism for the production of the initial answer for both algorithms is the same.

|             |             |             |                                 |             |
|-------------|-------------|-------------|---------------------------------|-------------|
| $Q_1^{max}$ | $Q_2^{max}$ | $Q_3^{max}$ | $\cdot \quad \cdot \quad \cdot$ | $Q_n^{max}$ |
|-------------|-------------|-------------|---------------------------------|-------------|

**Fig. 1** The initial answer vector

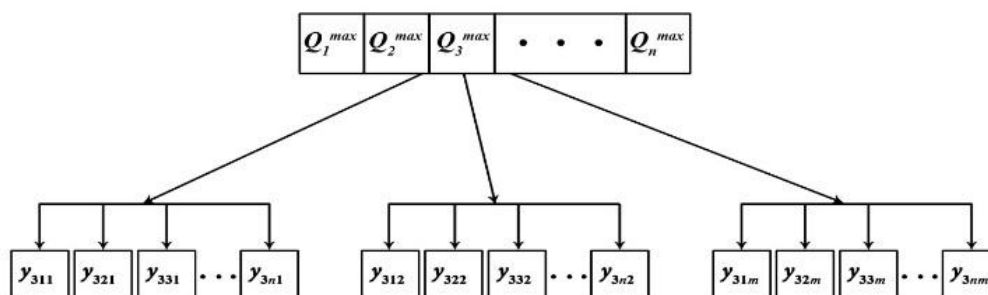
After producing the initial answer, answers are adjusted if needed. Then  $y_{jit}$  is valued according to  $Q_j^{max}$ , therefore objective functions are calculated based on the calculated initial amounts for  $Q_j^{max}$  and  $y_{jit}$ .

**Table 1** single-objective solution with software one to five

| LP-M solution with software 5                         | LP-M solution with software 4 | LP-M solution with software 3 | LP-M solution with software 2 | LP-M solution with software 1 | Single objective solution |
|-------------------------------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|---------------------------|
| The resulted answer for the first objective function  |                               |                               |                               |                               |                           |
| 1696000                                               | 1346200                       | 838400-                       | 1346200                       | 4416000-                      | 9438000                   |
| The resulted answer for the second objective function |                               |                               |                               |                               |                           |
| 47.318                                                | 18.76                         | 47.318                        | 18.760                        | 25.857                        | 6.988                     |
| The resulted answer for the third objective function  |                               |                               |                               |                               |                           |
| 2.45                                                  | 0                             | 0                             | 0                             | 109.270                       | 0                         |

**Table 2** solution time change trend in the first objective function

| Number of periods                       |                              |                              |                             |   | Number of products |
|-----------------------------------------|------------------------------|------------------------------|-----------------------------|---|--------------------|
| 5                                       | 4                            | 3                            | 2                           |   |                    |
| 11 seconds                              | 27 seconds                   | 32 seconds                   | 8 seconds                   | 2 |                    |
| 38 seconds                              | and minute 1<br>36 seconds   | 2 minutes and 16<br>seconds  | 14 seconds                  | 3 |                    |
| 37 minutes and 9 seconds                | 11 minutes and 26<br>seconds | 5 minutes and 29<br>seconds  | 3 minutes and 34<br>seconds | 4 |                    |
| 40 minutes and 14 seconds               | 15 minutes and 32<br>seconds | 3 minutes and 7<br>seconds   | 4 minutes and 5<br>seconds  | 5 |                    |
| 2 hours and 2 minutes and 52<br>seconds | 28 minutes and 43<br>seconds | 15 minutes and 58<br>seconds | 4 minutes and 30<br>seconds | 6 |                    |

**Fig. 2** Formation of allocation vectors from initial answer

## 5.4 Multi-objective genetic algorithm with Non-Dominated Sorting II

In 2001, Deb created multi-objective genetic algorithm with Non-Dominated Sorting by adding two operators to typical single-objective genetic algorithm. To study more about the

genetic algorithm, see the resources provided in this field [22]. In this study, Arithmetic Intersection Procedure is also used to create new Offspring.

### 5.5 Differential evolution algorithm

A differential evolution algorithm was presented by Storn and Price (1995) to optimize continuous spaces [23]. Of course, later versions of the algorithm were also provided for optimization in discrete spaces. In addition to general similarities of this algorithm to other production process evolutionary algorithms, the answer is quite unique in this algorithm.

### 5.6 The proposed algorithm parameter setting

In the meta-heuristic solution methods, parameters setting is created to improve the quality of answer and to increase the speed of Imperative solution. One of the most widely used and low-cost approaches to set parameters is to design tests. So here we use Taguchi method to set the parameters.

### 5.7 Setting algorithm parameters with Taguchi method

20 items were chosen to perform problem testing. In both algorithms, 4 parameters were considered in two levels to set. The test was repeated 10 times for each design. In each 10 tests, the indicators mean is considered as the amount of that indicator in the target test plan, and after normalization of indicators amount, the combination of them is considered as answer variable. The indicators which were taken into account to adjust the algorithm parameters include the variety and number of Pareto answers. According to the structure of criteria, the answer variable was considered as larger-better. Table (3-4) shows the optimum values obtained for each parameter.

### 5.8 Solving medium size example with meta-heuristic algorithms

After setting the parameters of target algorithms, an example with 56 items and 12 periods were selected to solve. Criteria which determine the superiority of algorithms include problem solving time, the number of Pareto answers, spacing and variety of responses generated by each algorithm. To ensure the sustainability of the answers of the problem, each algorithm is solved 35 times and the resulted numbers for the criteria are used in these 35 times. The percentage of superiority of each algorithm for each criterion is shown in the table (4-4). To prove the superiority of the algorithms for any indicator, these empirical evidences are required to be tested in statistical tests.

**Table 3** optimal values of meta-heuristic algorithms parameters levels

| The optimum value of parameters in differential evolution algorithm |              |                                    | The optimum value of parameters in multi-objective genetic algorithm with non-dominant sorting |              |                                    |     |
|---------------------------------------------------------------------|--------------|------------------------------------|------------------------------------------------------------------------------------------------|--------------|------------------------------------|-----|
| Value                                                               | Display Type | Name of parameter                  | Value                                                                                          | Display Type | Name of parameter                  | Row |
| 50                                                                  | npop         | Population size                    | 60                                                                                             | Npop         | Population size                    | 1   |
| 40                                                                  | maxit        | The maximum number of performances | 70                                                                                             | Maxit        | The maximum number of performances | 2   |
| 0.9                                                                 | Pc           | Probability of crossover           | 0.3                                                                                            | Pm           | The possibility of mutation        | 3   |
| 0.9                                                                 | F            | Length of mutation                 | 0.9                                                                                            | Pc           | The Probability of crossover       | 4   |

### 5.9 Means comparison test

In this section, means of indicators are tested for both communities.

The test evaluates the assumption of the equality of the means of both communities at the confidence level of 95 percent. Calculations were performed with software Minitab 17 and the results are shown in the table (4-5).

**Table 4** The result of meta-heuristic algorithms comparison with different criteria

| Solution Time                        | 100 percent superiority of differential evolution algorithm                                                            |
|--------------------------------------|------------------------------------------------------------------------------------------------------------------------|
| Name of Criterion                    | Result                                                                                                                 |
| Solution Time Variance               | superiority of differential evolution algorithm                                                                        |
| The number of Pareto Answers         | 91 percent superiority of multi-objective genetic algorithm with non-dominant sorting                                  |
| Spacing Index                        | 83 percent superiority of multi-objective genetic algorithm with non-dominant sorting                                  |
| The Resulted Answers diversity index | Approximately equal but multi-objective genetic algorithm with dominant sorting had better performance in 54% of cases |

As shown in Table (4-5) the assumption of the equality of the means is rejected for problem solving time, the number of Pareto answers and spacing. But about the diversity indicator, the test statistic is included in the range and also P-value is higher than 0.05, so there is no reason to reject the null hypothesis.

**Table 5** The results of statistical tests

| P-Value | T-Value | confidence interval |              | Sample standard deviation | Sample average | Number of samples | algorithm | Index                    |
|---------|---------|---------------------|--------------|---------------------------|----------------|-------------------|-----------|--------------------------|
| 0.000   | 334.66  | 96.734              | 97.896       | 1.38                      | 168.34         | 35                | NSGA-II   | Solution time            |
|         |         |                     |              | 1.03                      | 71.03          | 35                | DE        |                          |
| 0.000   | 7.4     | 12.37               | 21.52        | 10.2                      | 55.8           | 35                | NSGA-II   | number of Pareto answers |
|         |         |                     |              | 8.87                      | 38.9           | 35                | DE        |                          |
| 0.000   | 4.1-    | -86419316595        | -29740500918 | 50077284296               | 170383000000   | 35                | NSGA-II   | Spacing                  |
|         |         |                     |              | 67281620625               | 228463000000   | 35                | DE        |                          |
| 0.276   | 1.1     | -188652105585       | 651297954449 | 908781000000              | 13293600000000 | 35                | NSGA-II   | diversity                |
|         |         |                     |              | 850654000000              | 13062300000000 | 35                | DE        |                          |

**Table 6** Review of the algorithm performance

| Superior algorithm  | Superiority Criterion | Index                    |
|---------------------|-----------------------|--------------------------|
| DE                  | The less, the better  | Solution time            |
| NSGA-II             | The more, the better  | number of Pareto answers |
| NSGA-II             | The less, the better  | Spacing                  |
| Approximately equal | The more, the better  | Diversity                |

After evaluating the assumption of equality of means of communities, superiority determination of algorithms in each indicator is performed using Box graphs by software Minitab. The results in Table (4-6) show that the genetic algorithm with Non-Dominated Sorting II is better for solving this problem.

### 5.10 Discussion and evaluation of the second objective function's results

In models without substitution, the shortage has cost for the system. So lower shortage is better. But in models with substitution, it's not the case. Indeed, more profit and more shortage are better than less profit and less shortage and has less risk.

## 6 Conclusion

Existing a relation between the products items will lead to improved results. A part of this improvement is a result of reduced maintenance cost. By substitution of items, the profit is increased and the expired items are decreased. Single objective models can maximize the profit, but cannot consider caused dissatisfaction by substitution. This causes losing

customers in long-term and reducing profits. Also, in models with substitution, the higher risk of shortage does not necessarily mean facing a higher shortage. However, more profit with the higher risk of shortage is better than higher shortage can be compensated with substitution.

### Footnotes

- 1-Lingo
- 2- GAMS
- 3- Non-Dominated Sorting Genetic Algorithm-II(NSGA-II)
- 4- Deb
- 5- Differential Evolution
- 6- Storn, R
- 7- Price, K.V
- 8- Taguchi
- 9- Minitab

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